Sem – IV Paper: CC- IX

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Merits and demerits of Liquid Drop Model:

Merits:

1. It explains the spherical shape of most nuclei.

2. It helps to predict the nuclear binding energy and also to assess how much is available for consumption.

3. It helps to explain the stability of nuclei.

4. The radioactive phenomenon can be explained with the help of this model.

5. This model explains the phenomenon of artificial radioactivity.

6. Nuclear fission can be understood on the basis of this model

Demerits

1. It is a crude model. It does not explain all the properties of the nucleus.

2. It does not explain the high stability of nuclei with the magic number.

3. The concept of pairing cannot be explained with this model.

4. The discontinuities in BE/ nucleon cannot be explained.

5. It is not successful in interpreting the breaking of nuclei in lighter elements.

Nuclear Shell Model and Magic Numbers:

There exist a number of points in favour of the shell model of the nucleus which are as follows:

- Just as inert gases, with 2, 10, 18, 36, 54, … electrons, having closed shells show high chemical stability, nuclei with 2, 8, 20, 50, 82 and 126 nucleons – the so called magic numbers-of the same kind (either proton or neutron) are particularly stable. The binding energy is found to be unusually high implying high stability which is reflected in high abundance of isotopes with these proton numbers and isotones with these neutron numbers. Nuclei both with Z and N equal to one of the magic numbers are said to be doubly magic.
- The number of stable isotopes (Z = const) and isotones (N = const) is larger with respective number of protons and neutrons equal to either of the magic numbers. E.g., Sn (Z=50) has 10 stable isotopes, C (Z=20) has 6 while the biggest group of isotone is at N=82, then at N=50 and N=20.
- The three naturally occurring radioactive series decay to the stable end product $^{208}Pb_{82}$ with Z = 82 and N = 126 indicating extra stable configuration of magic nuclei.

- The neutron absorption cross-section is low for nuclei with N = magic numbers like 50, 82 and 126, indicating reluctance of magic nuclei to accept extra neutrons in their completely filled shells.
- Isotopes like $^{17}O_8$, $^{87}K_{36}$ and $^{137}Xe_{54}$ are spontaneous neutron emitters when excited by preceding β decay. The isotopes have N=9, 51 and 83 respectively, i.e., N = (8+1), (50+1) and (82+1). One can interpret this loosely bound neutron as a valence neutron which the isotopes emit to assume some magic N-value for their stability.
- Electric quadrupole moment Q of magic nuclei is zero indicating spherical symmetry of the nucleus for closed shells. When Z-value or N-value is gradually increased from one magic number to the next, Q increases from zero to a maximum and then decreases to zero at the next magic number.
- The energy of α- or β- particles emitted by magic radioactive nuclei is larger.

All these experimental facts lend a strong support to shell structure of nuclei.

In nuclear physics, the **nuclear shell model** is a theoretical model proposed by Dmitry Ivanenko in 1932 to describe the atomic nucleus. This model assumes that each nucleon stays in a **well-defined quantum state** and each nucleon is considered as a **single particle** that moves independently in the **time-averaged field of the remaining (A-1) nucleons acting as a core**, and is confined to its own orbit completing several revolutions before being disturbed by others by way of collisions. It must be noted that this model is based on the **Pauli exclusion principle** to describe the structure of the nucleus in terms of energy levels.

The **nuclear shell model** is partly analogous to the atomic shell model which describes the arrangement of electrons in an atom, in that a filled shell results in greater stability. When adding nucleons (protons or neutrons) to a nucleus, there are certain points where the binding energy of the next nucleon is significantly less than the last one. This observation, that there are certain magic numbers of nucleons (**2, 8, 20, 28, 50, 82, 126**) which are more tightly bound than the next higher number, is the origin of the shell model.

The shells for protons and for neutrons are independent of each other. Therefore, "magic nuclei" exist in which one nucleon type or the other is at a magic number, and "doubly magic nuclei", where both are magic number.

In order to get these numbers, the nuclear shell model starts from an average potential with a shape something between the square well and the harmonic oscillator. To this potential, a spin orbit term is added. Even though, the total perturbation does not coincide with experiment, and an empirical spin orbit coupling must be added with at least two or three different values of its coupling constant, depending on the nuclei being studied.

In comparison to atomic shell model, the **atomic nucleus** governed by two different forces. The residual strong force, also known as the nuclear force, acts to hold neutrons and protons together in nuclei. In nuclei, this force acts against the enormous repulsive electromagnetic force of the protons. The term residual is associated with the fact that it is the residual of the fundamental strong interaction between the quarks that make up the protons and neutrons. The strong interaction is very complicated interaction, because it significantly varies with distance. At distances comparable to the diameter of a proton, the strong force is approximately 100 times as strong as electromagnetic force.

With the enormous strong force acting between individual nucleons and with so many nucleons to collide with, how can nucleons orbit a central potential without interacting? This problem is explained by the **Pauli exclusion principle**, which states that two fermions cannot occupy the same quantum state. In other words, the interaction will not occur, if the higher energy shells are fully occupied and the energy imparted to the nucleon during the collision is insufficient to promote the nucleon to an unfilled orbit. As a result, the nucleons orbit becomes independent of one another. The nuclear shell model was able to describe many phenomena like the **magic numbers**, the ground state spin and parity etc.

Modelling the Shell Structure:

Whereas in atomic physics we solve the Coulomb force problem to get the shell structure, we expect that in nuclei the potential is more attractive in the centre, where the density is highest, and less attractive near the surface. There is no reason why the attraction should diverge anywhere, and we expect the potential to be finite everywhere. One potential that satisfies these criteria, and can be solved analytically, is the Harmonic oscillator potential. In terms of Schrodinger's equation, each nucleon thus moves in the same potential V(r), the average harmonic oscillator potential so that $V(r) = 1/2m\omega^2r^2$. The Schrodinger equation then becomes

$$-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi(\vec{r})+\frac{1}{2}m\omega^2r^2\psi(\vec{r})=E\psi(\vec{r}) \qquad (1)$$

Where m is the mass of the nucleon and E the energy eigenvalues,
The solution of equation (1) is given by $E_n = (n+1/2)\hbar\omega$ (2)

Where n = oscillator quantum no = 0, 1, 2….so that in the harmonic oscillator model, all the energy eigenstates are equally spaced. The wave function Ψ has both angular and radial part.

Each nucleon is supposed to have an orbital angular momentum $\boldsymbol{l}$ and $|\boldsymbol{l}| = \sqrt{l(l+1)}\,\hbar$ where l =0, 1, 2, …, the nuclear orbital quantum no. Each nucleon has also spin angular quantum number $\boldsymbol{s}$ and $|\boldsymbol{s}| = \sqrt{s(s+1)}\,\hbar$ where s = ½ and behaves as an independent particle subject to Pauli's exclusion principal that no two identical nucleons can be in the same quantum state. Again, as suggested by Mayer, Jensen and others, there is a strong interaction (coupling) between the orbital and the intrinsic spin angular momenta of each nucleon. The quantum mechanical rules for angular momenta dictate that total angular momentum $j\hbar$ formed by vector addition of orbital angular momentum $l\hbar$ and spin $s\hbar$ must be such that j is restricted to the following two values: $j = l+1/2$ and $l -1/2$.

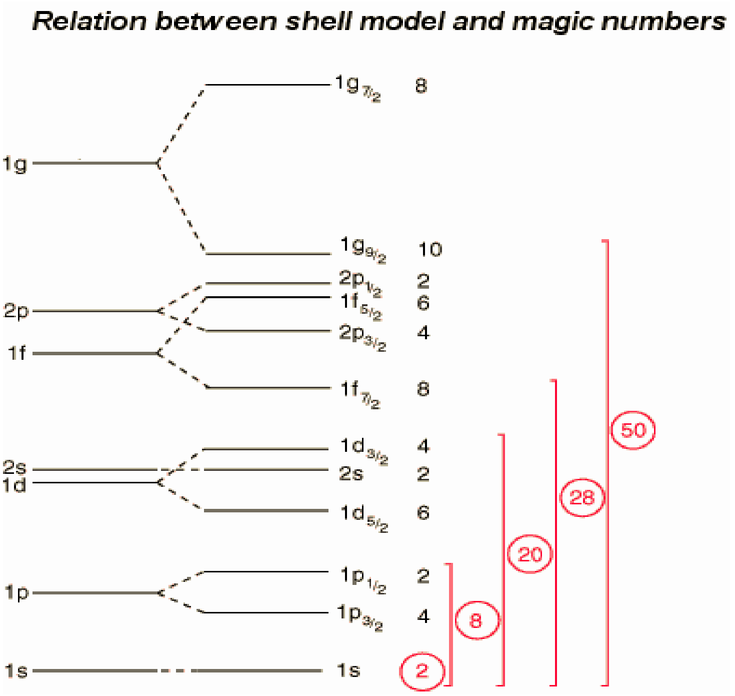
Thus a different energy is associated with each of the two j-levels and each nucleonic energy level with a given l splits into two sub-shells, except for $l = 0$, when j has only one value ½. The level $j = l+1/2$ corresponds to $\boldsymbol{s}$ and $\boldsymbol{l}$ parallel to each other and $j = l-1/2$ corresponds to $\boldsymbol{s}$ and $\boldsymbol{l}$ anti-parallel to each other. Empirically, it is found that the nuclear energy level with higher j always lie below that with smaller j. So, $j = l+1/2$ sub-level has a lower energy than $j = l-1/2$ sub-level, the former giving a more tightly bound nucleonic state. The separation between two sub-levels with a given l is rather large and increases rapidly with l.

To designate the nucleonic states, spectroscopic notation of atomic physics is followed. Each sub-level can have a maximum of $(2j+1)$ nucleons of the same kind, for a given j. So, it can be house $(2j+1)$ protons and $(2j+1)$ neutrons. Nucleons are designated with n-values followed by spectroscopic notation of l-values (s, p, d, f, …. for l = 0, 1, 2, 3, …). For instance, for $l = 0$, $j = l+1/2$ = ½ and the number of nucleons in the level = $2j+1$ = 2x1/2 =2 and the state is designated as $1s_{1/2}$. Similarly, for $l = 1$, $j = l+1/2 = 1+1/2 = 3/2$ and $j = l -1/2$ = ½. The number of nucleons in the two sub-shells are thus (2x3/2+1) = 4 and (2x1/2+1) = 2. The two substates are designated as $1p_{3/2}$ and $1p_{1/2}$ respectively. So, the total number of nucleons in this level =

4+2 = 6 giving the progressive total of 8 nucleons, and so it is going on. Thus, we can predict the completed shells and the corresponding total number of nucleons.

We can imagine ourselves building a nucleus by adding protons and neutrons. These will always fill the lowest available level. Thus, the first two protons fill level zero, the next six protons fill level one, and so on. As with electrons in the periodic table, protons in the outermost shell will be relatively loosely bound to the nucleus if there are only few protons in that shell, because they are farthest from the centre of the nucleus. Therefore, nuclei which have a full outer proton shell will have a higher binding energy than other nuclei with a similar total number of protons. All this is true for neutrons as well. This means that the magic numbers are expected to be those in which all occupied shells are full.

By Pauli's principle there can be no more than one proton and one neutron in each orbit, i.e., with the same quantum number (n, l, j, m). If we now consider nuclei in their lowest energy states, we expect the orbits to fill in order of increasing energy as with electrons in atoms. The $^{4}He_{2}$ nucleus with 2 protons and 2 neutrons will have all n=1 orbits full and others empty. A nucleus of this type (all shells either completely filled or completely empty) is called closed shell nucleus. We have closed shell nuclei like $^{16}O_{8}$, $^{40}Ca_{20}$, $^{48}Ca_{20}$, $^{208}Pb_{82}$ analogous to the inert gas structure of He, Ne, A, Kr, etc atoms.


Relation between shell model and magic numbers

- 1st shell: 2 states ($n = 0, j = \frac{1}{2}$).
- 2nd shell: 6 states ($n = 1, j = \frac{1}{2}$ or $\frac{3}{2}$).
- 3rd shell: 12 states ($n = 2, j = \frac{1}{2}, \frac{3}{2}$ or $\frac{5}{2}$).
- 4th shell: 8 states ($n = 3, j = \frac{7}{2}$;
- 5th shell: 22 states ($n = 3, j = \frac{1}{2}, \frac{3}{2}$ or $\frac{5}{2}$; $n = 4, j = \frac{9}{2}$).
- 6th shell: 32 states ($n = 4, j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ or $\frac{7}{2}$; $n = 5, j = \frac{11}{2}$).
- 7th shell: 44 states ($n = 5, j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ or $\frac{9}{2}$; $n = 6, j = \frac{13}{2}$).
- 8th shell: 58 states ($n = 6, j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$ or $\frac{11}{2}$; $n = 7, j = \frac{15}{2}$).

and so on.

The magic numbers are then

- 2
- 8 = 2+6
- 20 = 2+6+12
- 28 = 2+6+12+8
- 50 = 2+6+12+8+22
- 82 = 2+6+12+8+22+32
- 126 = 2+6+12+8+22+32+44
- 184 = 2+6+12+8+22+32+44+58

and so on. This gives all the observed magic numbers, and also predicts a new one (the so-called island of stability) at the value of 184 (for protons).

Success and limitations of Shell Model:

Some of the successes are

- It explains very well the existence of magic numbers and the stability and high binding energy on the basis of closed shells.
- The shell model provides explanation for the ground state spins and magnetic moments of the nuclei. The neutrons and protons with opposite spins pair off so that the mechanical and magnetic moment cancel and the odd or left out proton or neutron contributes to the spin and magnetic moment of the nuclei as a whole.
- Nuclear isomerism, i.e., existence of isobaric, isotopic nuclei in different energy states of odd-A nuclei between 39-49, 69-81, 111-125 has been explained with shell model by the large difference in nuclear spins of isomeric states as their A-values are close to the magic numbers.

Some of the limitations of the shell model are:

- The model does not predict the correct value of spin quantum number I in certain nuclei, e.g., $^{23}Na_{11}$ where the predicted value is I=5/2, the correct value is 1/2.
- The following four stable nuclei $^{2}H_{1}$, $^{6}Li_{3}$, $^{10}B_{5}$ and $^{14}N_{7}$ do not fit into this model.
- The model can not explain the observed first exited states in even-even nuclei at energies much lower than those expected from single particle excitation. It also fails to explain the observed large quadrupole moment of odd-A nuclei, in particular of those having A-values far away from the magic numbers.
- If all inter-nucleon couplings are ignored, the model is called single particle shell model. If, however, couplings are considered, it is known as independent particle shell model.